INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, Second Semester, 2005-06
Statistics - II, Midterm Examination, March 3, 2006

(10) 1. Let $X_1, \ldots, X_n$ be a random sample from the following distribution:

x	-1	0	1
$P_\theta(X = x)$	θ_1	θ_2	$1 - \theta_1 - \theta_2$

where $\theta_i > 0$, $i = 1, 2$ and $\theta_1 + \theta_2 < 1$.
(a) Find minimal sufficient statistics for (θ_1, θ_2). Is it complete?
(b) Find the maximum likelihood estimate of θ_2.

(8) 2. Suppose $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ are independent random samples, respectively, from $N(\mu, \sigma^2)$ and $N(2\mu, 2\sigma^2)$, where $-\infty < \mu < \infty$, $\sigma^2 > 0$.
Find minimal sufficient statistics for (μ, σ^2). Is it complete?

(8) 3. Suppose X_1 and X_2 are two i.i.d. observations from Binomial(n, p), $0 < p < 1$, n known. Let $\theta = p^n$. Find the UMVUE of θ.

(12) 4. For observations $y_1, \ldots, y_n$, consider the linear model

$$y_i = \beta x_i + \epsilon_i, \quad i = 1, \ldots, n,$$

where x_i is the value of a co-variate corresponding to y_i and ϵ_i are i.i.d. errors having the $N(0, \sigma^2)$ distribution with β, σ^2 unknown.
(a) Show that the distribution of $y_1, \ldots, y_n$ belongs to k-variate exponential family. Find k.
(b) Find minimal sufficient statistic for (β, σ^2). Is it complete?
(c) Find the least squares estimate of β.
(d) Find the MLE of (β, σ^2).

(12) 5. Let $X_1, \ldots, X_n$ be a random sample from Poisson(λ), $\lambda > 0$.
(a) Find the Fisher information of λ contained in the random sample.
(b) Find the Cramer-Rao lower bound on the variance of an unbiased estimator of $\exp(-\lambda)$.
(c) Find the UMVUE of $\exp(-\lambda)$. Does it attain the lower bound given in (b) above?